2014 **World Mathematics Team Championship**
Junior Level
Team Round · Problems

1. Observe the pattern of the following sequence of numbers:

$$1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \cdots$$

Which term gives the number $\frac{11}{26}$?

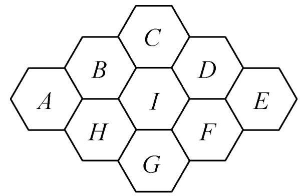


Fig. 1

2. How many ways can the number 14 be written as a sum of prime numbers? ($3+11$ and $11+3$ are considered the same)

3. The Fig. 1 is composed of nine identical regular hexagons each of edge length 1. Three neighboring (each has at least one common edge with another one) hexagons are taken out so that the remaining figure is not disconnected and has the same perimeter as the original figure. Which three hexagons can be taken out?

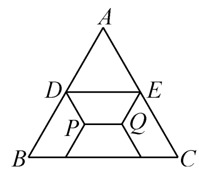


Fig. 2

4. Triangle ABC in the Fig. 2 is an equilateral triangle where D and E are midpoints of AB and AC, respectively. Trapezoid $DECB$ is divided into four smaller isosceles trapezoids of identical shape and area. How many times is the perimeter of $\triangle ABC$ large than the perimeter of the isosceles trapezoid $DEQP$?

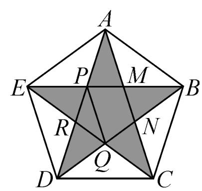


Fig. 3

5. How many integers a can satisfy the inequality $\frac{4}{11} < \frac{a}{2014} < \frac{5}{12}$?

6. Randomly select three distinct numbers from {1, 2, 3, 4, 5, 6} to form a 3-digit number. How many different quotients are possible if the sum of all such possible 3-digit numbers formed is divided by the different sums of their three digits?

7. As shown in the Fig. 3, $ABCDE$ is a regular pentagon and the area of quadrilateral $ACQP$ is 1. Find the area of the 5-point star $AMBNC\cdots PA$ (the shaded region).

8. As shown in the Fig. 4, $ABCD$ is a square where square $EBFP$ and square $MPND$ share a common vertex P and have areas of 64 and 16, respectively. Suppose the two vertices J and K of square $IJHK$ are the intersections of the two main diagonals of $EBFP$ and $MPND$, respectively. Find the area of square $IJHK$.

9. Suppose the side length of a square is a prime number a that is less than 20 and the side length of an equilateral triangle is a natural number b. If the perimeter of this square is 10 longer than the perimeter of this equilateral triangle, find the number of possible values for b.

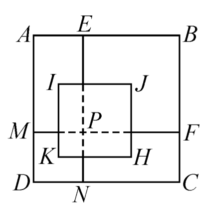


Fig. 4

10. Suppose x is a prime number and y is an integer. If $\frac{1}{xy} + \frac{2}{xy} + \cdots + \frac{55}{xy}$ is an integer, what is the maximum possible value for y?

11. Suppose numbers A and B have only prime factors of 2 and 5 and their Greatest Common Divisor (GCD) is 50. If A has 6 factors and B has 12 factors, find all possible $A+B$.

12. As shown in the Fig. 5, $ABCDEF$ is a regular hexagon. The shaded triangle has vertices at the intersection of AD and FC and the midpoints of AC and EC. The area of the shaded triangle is 6. Find the area of $ABCDEF$.

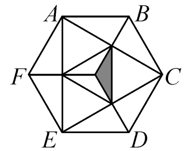


Fig. 5

13. How many fractions among $\frac{1}{2014}, \frac{2}{2014}, \frac{3}{2014}, \cdots, \frac{2012}{2014}$, and $\frac{2013}{2014}$ are simplified fractions?

14. Nine points can be used to construct 8 straight lines with each line passing through exactly 3 points (see Fig. 6) or 9 straight lines (see Fig. 7) or even 10 straight lines with each line passing through exactly 3 points (see Fig. 8). What is the maximum number of straight lines can be constructed using 10 points so that each line passes through exactly 3 points?

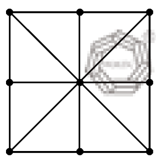


Fig. 6

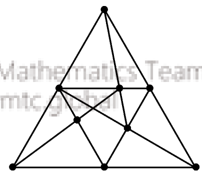


Fig. 7

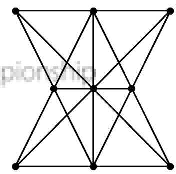


Fig. 8

15. Suppose there are 13 50 - passenger buses for a rental of \$800 per bus and 20 40 - passenger buses for a rental of \$680 per bus. If there is a total of 720 passengers, in order to minimize the total transportation cost, how many 40 - passenger buses should be used?

16. Given a number that has finite number of decimals and satisfies the condition that if its decimal point is moved to the right by a certain number of positions, the new number is larger than the original number by 9.999. Find the sum of all numbers that satisfy this condition.

17. Suppose N is a natural number with distinct digits and each digit is also its factor. Find the value of largest such number N.

18. Suppose three people A, B, and C are walking in the same direction, each at a constant speed, along a straight line. In the beginning, their positions are as shown in Fig. 9. After a certain amount of time, their positions are as shown in Fig. 10. As they continue walking with their original constant speed, A is finally positioned in the middle and is equal distanced from B and C. Find the distance between B and C at that time (all distances are measured in meters).

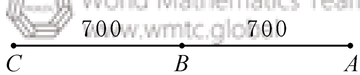


Fig. 9

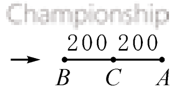


Fig. 10

19. Consider a cube with each of its six sides painted a different number from 1, 2, 3, 4, 5, and 6. Different views of that cube are shown in the Fig. 11. Suppose 12 of such cubes are put

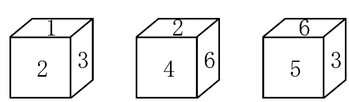


Fig. 11

together to form rectangular solids of different configurations. For each configuration, add up all the numbers on the outside of the solid (including those on the bottom surface). What is the smallest sum among all the configurations?

20. The Fig. 12 has 12 1×1 small squares and a total of 20 vertices (some squares share common vertices). Among the triangles that can be formed by using three points from these 20 points, how many of them have an area of 2?

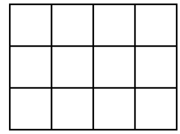


Fig. 12

Team Round Answers

1. 312.
2. 10.
3. H, G, F or B, C, D.
4. 2.4.
5. 107.
6. 1.
7. 2.
8. 36.
9. 3.
10. 770.
11. 6300 or 550 or 250.
12. 144.
13. 936.
14. 12.
15. 3.
16. 1.213.
17. 9867312.
18. 560.
19. 60.
20. 154.

Relay Round · Problems

First Round

1A. How many triangles are in the Fig. 1?

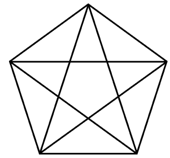


Fig. 1

1B. Let $T=$ TNYWR (The Number You Will Receive) and let $S=3T$.

Find the sum of all positive integers less than S that have odd number of factors.

Second Round

2A. The three numbers 277, 362, and 515 share a common characteristic in that they have a common remainder R when they are divided by a common divisor D not equal to 1. Find $(D-R)$.

2B. Let $T=$TNYWR (The Number You Will Receive). Suppose a circle of radius 8 and center O and a square of side length of T and has one of its vertices at O as shown in the Fig. 2.

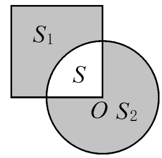


Fig. 2

Let $S_1=$area of the region inside the square but outside the circle.

$S_2=$ area of the region inside the circle but outside the square

Find S_2-S_1. (Use $\pi=3$)

Third Round

3A. Suppose $\begin{array}{r} 1WMTC \\ \times \quad\quad 3 \\ \hline 3MTC1 \end{array}$ where each letter represents a distinct digit. Find $W+M+T+C$.

3B. Let $T=$TNYWR (The Number You Will Receive) and let $S=T-19$. uppose there are N 2-digit numbers $\overline{ab}$ that satisfy $a-b=S$.

Find the units digit of the number $\underbrace{N \cdot N \cdot N \cdot N \cdots N}_{100N's}$. (product of 100 N's).

Relay Round Answers

First Round	Second Round	Third Round
1A. 35.	2A. 12.	3A. 22.
1B. 385.	2B.48.	3B. 1.

Individual Round • Problems

First Round

1. As shown in the Fig. 1, $AE=4$, $ED=3$, $BC=6$, and $DC=9$. Find the area of $\triangle ABD$.

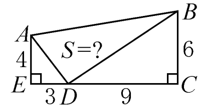


Fig. 1

2. Given that both 4-digit numbers $\overline{10ab}$ and $\overline{ba01}$ are perfect squares. Find $\overline{ab}$.

3. An opera house has 27 rows of seats. Each row has 2 more seats than the row immediately in front of it. If the 14^{th} row has 60 seats, how many seats does this opera house have?

4. Find the last digit (units digit) for the number

$$2\times 2014+3\times 2013+4\times 2012+\cdots+1008\times 1008.$$

Second Round

5. As in Fig. 2 $ABCDEF$ below is a regular hexagon. If the area of quadrilateral $ACDE$ is 8, find the area of this hexagon.

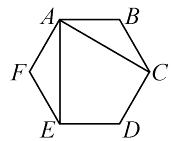


Fig. 2

6. Divide a cube with integer edge length into 153 smaller cubes in which 152 of them are cubes of edge length 1. Find the edge length of the original cube.

7. If each of the letters from a, b, c, d, and e represents a different number from 3, 4, 5, 6, and 7 and that $a+1+b=1+c+e=d+e+2$, find e.

8. As shown in Fig. 3, the area of isosceles right triangle ABC (with right angle at B and $AB=BC$) is 1. Now use the three edges AC, AB, and BC as sides to construct external squares $ACDE$, $AFGB$, and $BHIC$. Find the area of the hexagon $DEFGHI$.

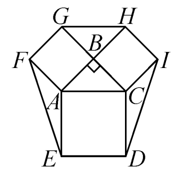


Fig. 3

Third Round

9. How many quadrilaterals does the Fig. 4 have?

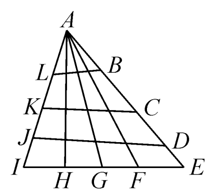
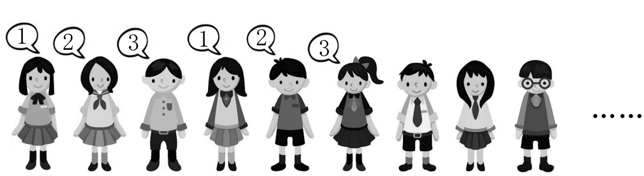


Fig. 4

10. Suppose A, B, and C are any three distinct digits from numbers 1 to 9 and $A+2B+3C=12$. If A, B, and C are used as digits, each used once and only once, to form a 3-digit number, how many such 3-digit numbers are larger than 321?

11. A group of students lined up in a straight line from left to right as they count off numbers 1, 2, 1, 2, 1, 2, … as shown in the first picture and then they would do the same again with numbers 1, 2, and 3 as shown in the second picture. If there are 5 people counted off the number "2" on both occasions, what is the most number of students this group can have?



12. A certain number of students engaging in rope jumping (skipping) contest are divided into three groups A, B, and C. Every contestant must belong to one and only one group. The average numbers of rope jumping performed by students in groups A, B, and C are 100, 80, and 70, respectively. If groups A and B are combined, their combined average is 85 times. If groups B and C are combined, the combined average is 76 times. What is the average when all three groups are combined?

Fourth Round

13. Suppose ABC is an isosceles right triangle with right angle at A and $AB=4$. Use A, B, and C as centers and AB as the radius to draw three arcs forming a figure as shown below. Find the area of the shaded portion of Fig. 5. (Use $\pi=3$)

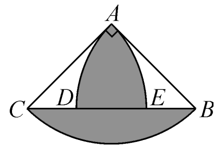


Fig. 5

14. The Fig. 6 is composed of $7n$ small 1×1 squares. Rectangles with edges parallel to the edges of these 1×1 squares can be constructed by connecting vertices of these squares. If there are exactly 154 such rectangles (excluding squares) with area 3, find n.

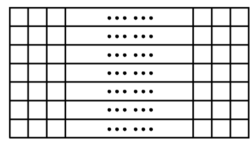


Fig. 6

Fifth Round

15. Consider the two circles as shown in the Fig. 7. If $\frac{2}{3}$ of the small circle and $\frac{4}{5}$ of the large circle are shaded and if the small circle has an area of 12, what is the area of the large circle?

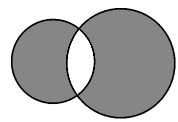


Fig. 7

16. As shown in Fig. 8, convex 4 - sided polygons have 2 diagonals, convex 5 - sided polygons have 5 diagonals, and convex 6 - sided polygons have 9 diagonals, and so on. How many sides do convex polygons have if there are 2015 diagonals?

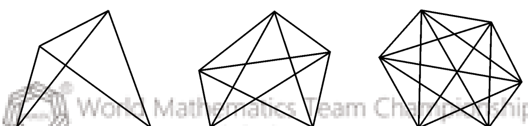


Fig. 8

Individual Round Answers

First Round

1. 27.
2. 89.
3. 1620.
4. 7.

Second Round

5. 12.
6. 6.
7. 4 or 7.
8. 12.

Third Round

9. 60.
10. 4.
11. 31.
12. 80.

Fourth Round

13. 8.
14. 14.

Fifth Round

15. 20.
16. 65.