2014 **World Mathematics Team Championship**
Intermediate Level
Team Round · Problems

1. Given that $\begin{cases} A+B+C=1, \\ B+C+D=2, \\ C+D+E=3, \\ D+E+F=4, \\ E+F+G=5, \\ F+G+H=6, \\ G+H+I=7, \end{cases}$ Find $A+E+I$.

2. A dart board has points labeled on different regions as shown in the Fig. 1. There are 30 different prizes for the winners and each prize is labeled by a whole number from 1 to 30. The game is played according to the following rules:

(1) Each player can throw the dart to the dart board three times,

(2) Darts outside the board get 0 point,

(3) Each player gets a prize that corresponds to the total number of points from these three throws.

How many of these 30 prizes will never be claimed by any player?

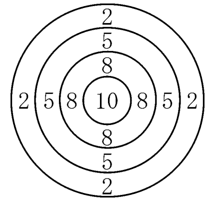

Fig. 1

3. If $ab\neq 1$ and $\begin{cases} 5a^2+1001a+1025=0, \\ 1025b^2+1001b+5=0, \end{cases}$ find the value for $\frac{a}{b}$.

4. As shown in the Fig. 2, square $ABCD$ has side length of 2. Using BC as diameter, construct a semi - circle O. Let F be the point on CD so that AF tangents to semi - circle O at point E. Find the length of ED.

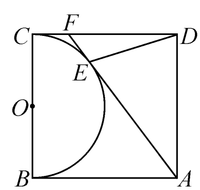

Fig. 2

5. If $\frac{x}{2x^2+9x+2}=\frac{1}{3}$, find the value for $\frac{x^2}{2x^4+x^2+2}$.

6. We know that it is Tuesday on 2014/11/11. Among the six dates listed below, which one (use its number to indicate) is not a Friday?

(1) 2014/2/2, (2) 2014/4/4, (3) 2014/6/6, (4) 2014/8/8, (5) 2014/10/10, (6) 2014/12/12.

7. Find the smallest integer t so that the set of inequalities of x $\begin{cases} \frac{x+1}{3}-t>2x \\ x^2-8x-9<0 \end{cases}$ has exactly 3 integer solutions.

8. As shown in the Fig. 3, Circle O has a radius of 20 and Circle O' tangents internally with Circle O at P and tangents externally at M with square $ABCD$ which is inside Circle O with its vertices A and B on Circle O and $AB\perp PM$. If $AB=24$, find

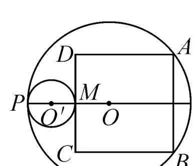

Fig. 3

the radius of Circle O'.

9. There are four book shelves in a small library which are labeled as #1, #2, #3, and #4 and they, respectively, hold 120, 135, 142, and 167 books. A day later, a number of books have checked out from each shelf. Another day later, 0, b, c, and d more books have checked out again from Shelves #1, #2, #3, and #4, respectively. At that time, all 4 shelves have the same number of books remained. If b, c, and d all $\geqslant 1$ and $b+c+d=a$, how many books remained on Shelf #1 after these two checkouts?

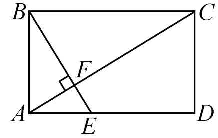

Fig. 4

10. Suppose b and c are two different real numbers. If r and s are roots of equation $x^2+bx+c=0$ and r and t are roots of equation $x^2+cx+b=0$, find $s+t$.

11. Consider the rectangle $ABCD$ as shown in the Fig. 4. Suppose E is on AD and that $CD=9$ and $AE:ED=3:5$ and $AC \perp BE$ at F. Find $BC:BA$.

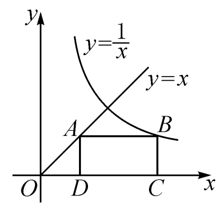

Fig. 5

12. Select n numbers from natural numbers 1 to 2014 with the condition that the difference of any two numbers from these n selected numbers cannot be 3. What is the maximum value for n?

13. Suppose rectangle $ABCD$ is located so that point A is on the straight line $y=x$, B is on the curve $y=\frac{1}{x}$, and points C and D are on the x-axis as shown in the Fig. 5. If $AB=2BC$, find the area of rectangle $ABCD$.

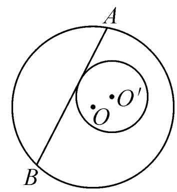

Fig. 6

14. Suppose Circle O has a radius of r and Circle O' has a radius of 11 and Circle O' is inside of Circle O with center O inside Circle O' as shown in the Fig. 6 on the right. If AB is a chord of Circle O that tangents Circle O' and $40 \leqslant AB \leqslant 48$, find OO'.

15. If $5a+4b+3c$ is divisible by 13 where a, b, and c are positive integers, find the remainder when $3a+5b+7c+2014$ is divided by 13.

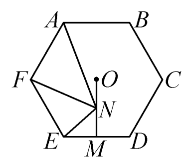

Fig. 7

16. As shown in the Fig. 7, O is the center of a regular hexagon $ABCDEF$ where $OM \perp DE$ at M and N is the midpoint of OM. If the area of $\triangle FAN$ is 10, find the area of hexagon $ABCDEF$.

17. As shown in the Fig. 8, $ABCD$ is a square with point E on CD. Extend BC and AE so they intersect at F. If the area of square $ABCD$ is the same as the area of $\triangle CFE$, find $\frac{CE}{ED}$.

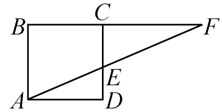

Fig. 8

18. Given two cubes with edge lengths of positive integers a and b. If the ratio between the sum of their volumes and the sum of their edge lengths is $27:1$, find $a+b$.

19. Suppose the parabola $C: y=\frac{1}{2}x^2+bx-1$ has three intersections with x and y coordinate axes. If these three intersections form a triangle and the area of this triangle increases 14 times when parabola C moves down 5 units, find b.

20. Suppose a ferry terminal's original schedule is to send out one ferry boat at every 8 minutes interval. Each ferry boat cruises the river and returns to the terminal in 80 minutes when it would meet another regularly scheduled departing ferry boat. The terminal has a ferry boats and this number of boats work perfectly with this schedule. Now, if two new ferry boats are added to the service, how many minutes would these additions shorten the original 8 minutes service interval?

Team Round Answers

1. 4.

2. 6.

3. 205.

4. $\frac{2\sqrt{10}}{5}$.

5. $\frac{1}{15}$.

6. (1)2014/2/2.

7. -4.

8. 6.

9. 36.

10. -1.

11. $\frac{2}{3}\sqrt{6}$.

12. 1008.

13. $\frac{2}{3}$.

14. 4.

15. 12.

16. 48.

17. $\sqrt{3}+1$.

18. 9.

19. $\frac{1}{2}$ or $-\frac{1}{2}$.

20. $1\frac{1}{3}$.

Relay Round • Problems

First Round

1A. Consider Circle O with radius 3 and Circle P with radius 7 where AB is a common external tangent to both circles as shown in the Fig. 1 with A and B being the points of tangency. If $OP=26$, find AB.

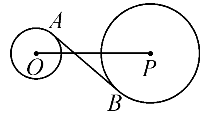

Fig. 1

1B. Let $T=$TNYWR (The Number You Will Receive).

Find the smallest 3 - digit number $\overline{abc}$ so that T is the remainder when the 9 - digit number $\overline{123456abc}$ is divided by37.

Second Round

2A. If the square of the sum of three distinct prime numbers is a 3 - digit number $\overline{aa5}$, find the product of these three prime numbers.

2B. Let $T=$TNYWR (The Number You Will Receive).

As shown in the Fig. 2, OAB is a quarter circle of radius T. Suppose X is the portion of arc AB that consists of all points P on arc AB such that $PA<T$ and $PB<T$. Find the length of arc X. (Use 3 for π)

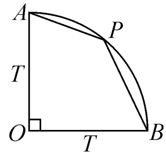

Fig. 2

Third Round

3A. Find the sum of all roots of equation $x^4-5x^3+5x^2+5x-6=0$.

3B. Let $T=$TNYWR (The Number You Will Receive).

As shown in the Fig. 3, triangle $\triangle ABC$ is a right triangle with $\angle ACB=90°$. Suppose quadrilaterals $ABDE$ and $BCFG$ are identical rectangles. If $AB=5$, $AC>BC$, and area of $S_{\triangle BDG}=T$, find area of $S_{\triangle BDC}$.

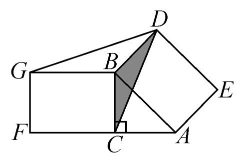

Fig. 3

Relay Round Answers

First Round

1A. 24.

1B. 111.

Second Round

2A. 105.

2B. 52. 5.

Third Round

3A. 5.

3B. $\frac{\sqrt{5}}{2}$.

Individual Round • Problems

First Round

1. If a is an integer and $a<\sqrt{2014}<a+1$, find $1+2+3+\cdots+a$.

2. If $\begin{cases} x-14\sqrt{xy}+\sqrt{y}=13, \\ y+16\sqrt{xy}+\sqrt{x}=17, \end{cases}$ find $\sqrt{x}+\sqrt{y}$.

3. As shown in the Fig. 1, $ABCD$ is a square with side length of 4. Suppose $ED=1$ and point M moves along side DC. Find the length of MC when the perimeter of $\triangle BME$ is at minimum.

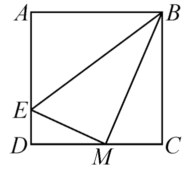

Fig. 1

4. If $y=ax^2-ax-8-2a>0$ holds when $4\leqslant x\leqslant 6$, find the range of values for real number a.

Second Round

5. Suppose $\overline{ab}$ is a 2 - digit number and $\overline{ba}-\overline{ab}=\overline{a0b}-\overline{ba}$, find $\overline{ab}$.

6. Six people standing in a circle as shown in the Fig. 2. They are playing a game of passing the ball. Each person who gets the ball will pass to a person who is not standing "next" to him. For example, B and F are standing "next" to A. In the beginning, A has the ball. What is the probability that the ball will pass back to A after three passes?

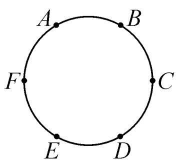

Fig. 2

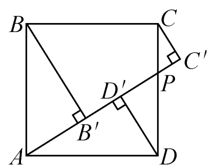

Fig. 3

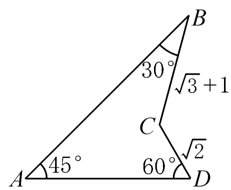

Fig. 4

7. As shown in the Fig. 3, point P is on the side CD of square $ABCD$ such that $AP=4$. Let BB', CC', and DD' be, respectively, distances from points B, C, and D to AP. If $BB'+CC'+DD'=6$, find the area of square $ABCD$.

8. Find the area of quadrilateral $ABCD$ as shown in the Fig. 4.

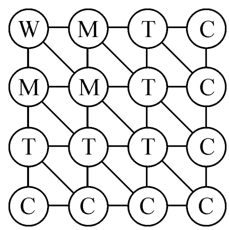

Fig. 5

Third Round

9. A beetle is crawling following the paths starting from the letter "**W**" on the upper left hand corner as shown in the Fig. 5. It stops after the 4^{th} letter. How many paths can the beetle take to crawl by the four letters "**WMTC**"?

10. If $x+y=2$ and $x^2+y^2=4$, find the value for $x^{2014}+y^{2014}$.

11. As shown in the Fig. 6, $\triangle ABC$ is an isosceles triangle with $\angle B=\angle C$ and E is a point on CB so that $\angle B=\angle C=\angle AEM$. If $BE=1$, $CE=2$, and $AC=4$, find AM.

12. Among the natural numbers from 1 to 154, find the sum of those that are relatively primes with 154.

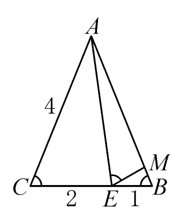

Fig. 6

Fourth Round

13. AS in Fig. 7, Suppose points A, B, and C are on the parabola $y=x^2$ so that $\triangle ABC$ is a right triangle with AB as its hypotenuse and parallel to the x-axis. If the area of $\triangle ABC$ is 2, find the y-coordinate of point C.

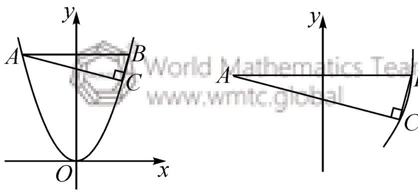

Fig. 7

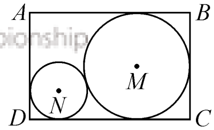

Fig. 8

14. AS in Fig. 8, Let $ABCD$ be a rectangle with $AB=18$ and $AD=12$. Suppose circle M tangents to sides AB, BC, and CD and circle N tangents to sides AD, DC, and externally tangents to circle M. Find the radius of circle N.

Fifth Round

15. Write down natural numbers from 1 to 50 in order to form another natural number $M = 123456789101112\cdots47484950$. If M is factored into prime factors, what is the highest power for the factor of 3?

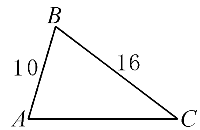

Fig. 9

16. Consider the triangle $\triangle ABC$ as in the Fig. 9. If $AB=10$, $BC=16$, and $\angle A=2\angle C$, find AC.

Individual Round Answers

First Round

1. 990.
2. 5.
3. $\frac{16}{5}$.
4. $a>\frac{4}{5}$.

Second Round

5. 16.
6. $\frac{2}{27}$.
7. 12.
8. $\frac{3\sqrt{3}+7}{2}$.

Third round

9. 15.
10. 2^{2014}.
11. $\frac{7}{2}$.
12. 4620.

Fourth Round

13. 3.
14. $24-12\sqrt{3}$.

Fifth Round

15. 1.
16. $15\frac{3}{5}$.