2013 World Mathematics Team Championship
Junior Level
Team Round • Problems

1. Let A, B, C, D, E, F, G, H, I, and J be an ordered set of prime numbers with a sum of 60. If any subset of 5 adjacent numbers from this set are mutually distinct, and has the same sum, how many different values can A + B take on?

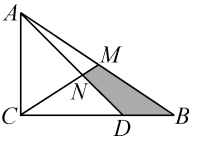


Fig. 1

2. Given right triangle $\triangle ABC$ with right angle at C as shown in Fig. 1. If $AC=10$, $BC=15$, $BD=5$, and $AM=MB$, find the area of the quadrilateral $DBMN$ (shaded area).

3. A kindergarten gives out apples to its students from three classes during a field trip. Suppose class A receives 2 less than $\frac{1}{3}$ of the total number of apples, class B receives 3 less than $\frac{1}{2}$ of what is left, and class C gets the remaining apples after A and B. If class C has more apples than class A and their difference is exactly $\frac{1}{30}$ of the original total number, what is the total number of apples in the beginning?

4. A given rectangle that is composed by $5 \times m$ small squares each painted with red or orange or yellow or green color. We call a rectangle "beautiful" if its four corners are painted with the same color. For Example, in the following colored 4×3 rectangle, the shaded area is a beautiful rectangle because its four corners are all red.

G	Y	Y
R	G	R
R	O	R
Y	R	G

R: Red
O: Orange
Y: Yellow
G: Green

Fig. 2

what is the smallest value for m so that there is always a beautiful rectangle within the $5 \times m$ rectangle grid?

5. Robots X and Y start at the same time from locations A and B, respectively, walking in opposite directions facing each other. As soon as they have reached the other location, they would turn around and walk back to the location where they came from. The speed of X is 12 meters per minute and the speed of Y is 16 meters per minute. If the distance between where they met the second time and where they met the third time is 80 meters, find the distance in meters between A and B?


Fig. 3

6. What time that is closest to 1 o'clock where the hour hand and minute hand are perpendicular to each other? (perpendicular: $\perp$, as show in the figure, at 3:00 o'clock, the hour hand and minute hand are perpendicular to each other.)

7. As in the Fig. 4, two satellites A and B are moving at same constant speeds counterclockwise around earth O on a circular paths using O as the center. If the proportion of the times it takes for A and B make one revolution around O is 1:8 and if satellites A and B start from the positions shown in the figure, how many times would they and earth O form a straight line during the time

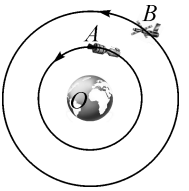


Fig. 4

when B makes one revolution around O?

8. The Fig. 5 has 60 squares with points A, B, C, and D on grid points as shown. Suppose a beetle starts from point A and crawls passing B and C on the way to point D. If this beetle is only allowed to crawl to the right or up from one grid point to another point. How many different paths available for this beetle to choose?

9. Consider the rectangle $EFGH$ as in the Fig. 6, where $GF=6$, $EF=4$, and the area of the shaded region is 7. Find the area of quadrilateral $ABCD$.

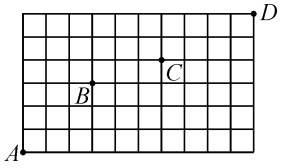


Fig. 5

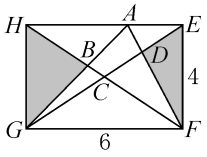


Fig. 6

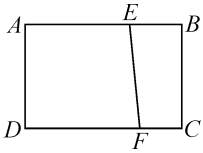


Fig. 7

10. Given a rectangle $ABCD$ as shown in the Fig. 7, where $AB=3$, $AD=2$, point E is on AB, $EB=\frac{1}{3}AB$, and point F is on DC. If proportion between the areas of trapezoids $EBCF$ and $AEFD$ is $3:7$, find the value for $FC:DF$. (Express answer in simplified fraction)

11. A and B are two circular disks that can rotate around their centers. Each disk surface is divided into 6 same sized sections and each section is labeled by a number from 1 to 6 as shown in the Fig. 8. A rotates counterclockwise at a speed of 2 revolutions per minute. B rotates clockwise at a speed of 3 revolutions per minute. As shown in the Fig. 8, the sections that have the number 1 on both disks A and B are closest to each other. Find the sum of the two numbers on A and B that are closest to each other after 1 minute and 20 seconds.

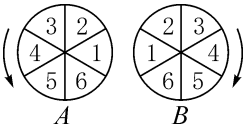
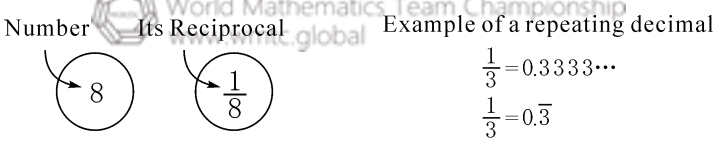


Fig. 8

12. Use natural numbers 1 to 9 once and only once as the tens and units digits for some numbers so that the sum of these numbers is 108. How many ways can this be done?

13. Consider there are 18 coins in which 7 of them are 10¢s, 6 50¢s, and 5 \$1's. How many ways there are in using 4 coins out from these 18 coins so that the total value is no more than \$1.80? (\$1=100¢)

14. Take two numbers randomly from natural numbers 1 to 49. Find the probability of both reciprocals of these two numbers are repeating decimals. (Express answer in simplified fraction)



15. Beijing, City A and City B are three cities with time difference. Suppose one way flight from any city to another city uses the same amount of time. Below is a table that shows the departure and arrival times (local times) of those flights.

	Date	Departure Time	Arrival Time
Beijing — City A	October 3	11 : 40	14 : 30
City A — City B	October 3	19 : 55	22 : 05
City B — City A	October 10	06 : 05	10 : 15
City A — Beijing	October 10	12 : 25	23 : 15

Question: When the plane arrives Beijing, what is the local time in City B?

16. Given a rectangular piece of paper. Cut the largest area square from this rectangle using one cut. Continue doing this 3 more times to each resulting figure after cutting. If the end result after these 4 cuts is exactly a square of side 1, find the sum of areas of all possible original rectangles.

17. To apply an operation to a simplified fraction means to add 1 to its numerator and subtract 1 from its denominator and simplify the resulting fraction. For example, $\frac{3}{11}$ becomes $\frac{2}{5}$ after applying the operation once.

$$\begin{matrix}\text{number} \\ \text{denominator}\end{matrix}\searrow\begin{matrix}3 \\ 11\end{matrix}\xrightarrow{\text{Operation}}\frac{3+1}{11-1}=\frac{4}{10}=\frac{2}{5}$$

If the sum of the numerator and denominator of a simplified fraction is 120 and it becomes 1 after applying the operation on it twice, how many simplified fractions have this kind of property?

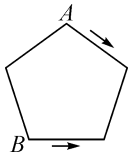


Fig. 9

18. As shown in the Fig. 9, A and B walk along the sides of a regular pentagon. A walks clockwise and around the pentagon once in 15 minutes. B walks counterclockwise and around the pentagon once in 20 minutes. If A and B start at the same time from the vertices that are indicated in the figure, how many minutes would it take for them to meet at one of the vertices for the first time?


Fig. 10

19. A basket has c number of chicken eggs and g number of pigeon eggs with $c:g=3:17$. Then additional chicken eggs are put into the basket. For every chicken egg putting into the basket, 5 pigeon eggs are taken out. This is considered one operation. After n operations, the proportion of chicken eggs to pigeon eggs is 2:3. Find the smallest possible value for n.

20. If a $7\times7\times7$ cube is cutting into many small cubes of edge length of 1 or 2 or 3, what is the least number of these little cubes can be obtained?

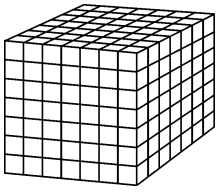


Fig. 11

Team Round Answers

1. 10.

2. $\frac{35}{2}$.

3. 180.

4. 41.

5. 140.

6. $12 : 49\frac{1}{11}$ (or $0 : 49\frac{1}{11}$).

7. 14.

8. 1200.

9. 1.

10. $\frac{4}{11}$, or $4 : 11$.

11. 6.

12. 254.

13. 6.

14. $\frac{703}{1176}$.

15. $18 : 15$.

16. 187.

17. 3.

18. 48.

19. 25.

20. 85.

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Relay Round • Problems

First Round

1A. As in the Fig. 1, a circle is divided by 5 line segments into 8 parts. If we paint these 8 parts so that neighboring parts have different colors, how many colors are required at least?

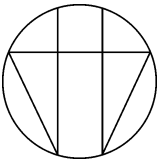

Fig. 1

1B. Let T = TNYWR (The Number You Will Receive).

Among the natural numbers from 1 to 10^T, how many of them have even number of factors? (10^T means multiply 10 T times and it is read as 10 to the T power.)

Second Round

2A. If the Least Common Multiple of 3 consecutive natural numbers is 1092, find the smallest of these three numbers.

2B. Let T = TNYWR (The Number You Will Receive).

Suppose the square $ABCD$ has a side length of T as shown in the figure below. Let A, D, C, and B be the centers and AB, DM, CQ, and BP be the respective radius to draw quarter circles such that M is on the extension of DA intersecting circle A, Q is on the extension of CD intersecting circle D, P is on the extension of BC intersecting circle C, and N is on the extension of AB intersecting circle B. Find the area of quadrilateral $MNPQ$.

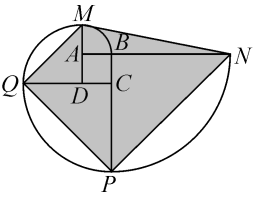

Fig. 2

Third Round

3A. In a Physical Education class, a coach prepared some basketballs and volleyballs. The number of basketballs is twice the number of volleyballs left after the boys checked out 10 volleyballs. Later, the number of volleyballs is twice the number of basketballs left after the girls checked out 30 basketballs. In total, how many basketballs and volleyballs did that coach have in the beginning?

3B. Let T = TNYWR (The Number You Will Receive).

A certain High Speed Rail train has 100 first class seats, 200 second class seats, and 400 third class seats and their ticket prices are \$2000, \$1500 and \$800, respectively. If $T\%$ of all tickets were sold, what is the difference between the maximum and minimum amounts received from ticket sales?

Relay Round Answers

First Round	Second Round	Third Round
1A. 2.	2A. 12.	3A. 70.
1B. 90.	2B. 2592.	3B. 197000.

Individual Round • Problems

First Round

1. Compute: $\left(\frac{2013}{2014}+\frac{20122013}{20142014}+\frac{201320132013}{201420142014}\right)\div\frac{2013201320132013}{2014201420142014}$.

2. If the product of two natural numbers is 1000 and their difference is 30, find their sum.

3. The mother's current age is 3 times the sum of ages of her two children. However, twenty two years later, her age is the sum of ages of her two children at that time. If the age difference of her two children is less than 5 but no less than 2, what are the ages of her children now?

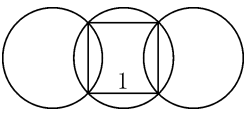

Fig. 1

4. As shown in the Fig. 1, the intersecting points of three identical circles form a square of length 1. Find the area covered by these three circles. (Use $\pi=3$)

Second Round

5. Among the families in a community, $\frac{2}{5}$ have air conditioning, $\frac{1}{3}$ don't have refrigerator, $\frac{1}{6}$ have neither air conditioning nor refrigerator, and 49 families have both air conditioning and refrigerator. How many families are there in the community?

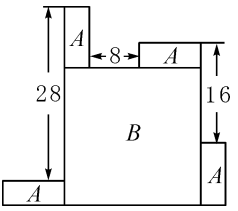

Fig. 2

6. The Fig. 2 is composed of 4 identical rectangles A and one square B. Find the area of a rectangle A.

7. As shown in the Fig. 3, A, B and C are part of a belt drive system. Assuming no slippage, B turns 3 rounds when A turns 4 rounds, and B turns 4 rounds when C turns 5 rounds. If the circumference of A is 1.5 meters, find the circumference of C in meters.

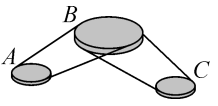

Fig. 3

8. A large piece of paper is divided into two portions of black and grey colors. Now place a piece of small paper half of the area of the large paper over the large paper as shown in the figure below. If the small paper covered $\frac{1}{4}$ of the black portion, and the uncovered black portion is $\frac{1}{4}$ of the area of large paper, find the simplified fraction $\frac{\text{the area of the covered grey portion}}{\text{the area of the large paper}}$.


Fig. 4

Third Round

9. In a photography contest, it was decided originally that the top 5 scored photographers get gold medal, and the next top 10 photographers get silver medals. However, it was decided later to move the last 2 gold medal winners to silver medals. This way, the average scores of both the new gold and silver medal winners would increase by 3 points. How much is the average score of the original gold medal winners more than the average score of the original silver medal winners?

10. As shown in the Fig. 5, the area of circle O is 1, AB is the diameter, point E is on CD, and $AB = 2BD = 2CD$. Find the area of the shaded portion.

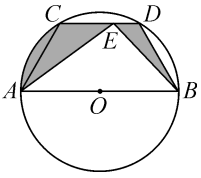

Fig. 5

11. Given 4 regions M, N, P, and Q as shown in the Fig. 6. Each region can be painted in one of four possible colors. If neighboring regions (such as M and N) cannot be painted with the same color, how many different ways these four regions can be painted?

12. Write natural numbers 1 to 2013 one after another in ascending order to obtain a long number 1234567891011…20122013. If divide this number by 9, what is the remainder?

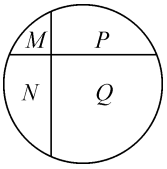

Fig. 6

Fourth Round

13. If the sum of all natural numbers between a prime number a and another prime number b is 280 and there is no prime number in between a and b, find $a + b$.

14. X and Y start at the same time going at constant speeds from locations A and B, respectively, toward each other. After they have met somewhere in between A and B, X continues to B. However, Y has two options:

(1) If Y continues to A, then Y will have to go another 45 kilometers to arrive in A after X had reached B;

(2) If Y returns to B with the same speed, then Y will have to go another 15 kilometers to arrive in B after X had reached B.

Find the distance in kilometers between A and B.

Fifth Round

15. A school has less than 300 students but both numbers for boys and girls are more than 100. If girls are grouped into 3 students a group or 4 or 5, there is always one student left over. If boys are grouped into 7 students or 8 students a group, each will be 3 students short. What is the most number of students this school can have?

16. Mary brings her daughter to swimming. Her daughter's swimming speed is $\frac{2}{3}$ of Mary's. Suppose the pool is 25 meters long and Mary and her daughter start their swimming at the same time from the same end. How many times will they meet each other after Mary had swum across the pool 20 round trips? (Do not count the first time when they started the swimming.)

Individual Round Answers

First Round

1. 3.

2. 70.

3. 4 and 7.

4. 4.

Second Round

5. 210.

6. 40.

7. 1.6.

8. $\frac{5}{12}$.

Third Round

9. 22.5.

10. $\frac{1}{6}$.

11. 84.

12. 3.

Fourth Round

13. 112.

14. 90.

Fifth Round

15. 290.

16. 33.