2012 **World Mathematics Team Championship**
Junior Level
Team Round • Problems

1. Given $\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{a}}}}}}=\frac{116}{607}$. Find a.

2. Each square below has a whole number less than or equal to 9 (includes 0). Starting with the first square on the left and take the sum of the three numbers in the three adjacent squares. Then start with the second square on the left and take the sum of the three numbers on the three adjacent squares. Follow this pattern, the sums are in order 6, 5, 3, 4, 6, 5, 3, 4, ⋯.

a	b	c	d	1	3	⋯

Then find the number inside the 1123rd square from the left.

3. Base 3 numbers can only have 3 possible digits: 0, 1, 2. For examples, 12_3, 201_3, 1122_3.
Any Base 3 number can be converted into a Base 10 number as follows:
Base 3 number: 12_3, corresponding Base 10 number:

$$1\times3^1+2\times3^0=3+2=5_{10}.$$

Base 3 number: 201_3, corresponding Base 10 number:

$$2\times3^2+0\times3^1+1\times3^0=18+0+1=19_{10}.$$

Base 3 number: 1122_3, corresponding Base 10 number:

$$1\times3^3+1\times3^2+2\times3^1+2\times3^0=27+9+6+2=44_{10}.$$

On the other hand, any Base 10 number can also be converted to a Base 3 number by using short division as follows:

```
3 | 44     Remainder
  3 | 14     ⋯ 2   ↑
    3 | 4    ⋯ 2   |
        1    ⋯ 1   |
```

Therefore, $44_{10}=1122_3$.

Consider the 5-digit Base 3 numbers which the digits read the same from left to right or from right to left. Find the sum of the largest and smallest among all such 5-digit Base 3 numbers. Write the sum both in Base 3 and in Base 10.

4. Suppose $n=\underbrace{222\cdots2}_{2012\ 2's}\times\underbrace{333\cdots3}_{2012\ 3's}\times9$. Find the sum of all the digits of n.

5. Suppose a and b are nonzero natural numbers and $\frac{a}{4}+\frac{b}{8}$ has an approximated value of 2.6. What is its exact value of this sum?

6. The Fig. 1 is a regular pentagon with vertices A, B, C, D, and E. Each of its interior angles is 108°. Connect the vertices and form its 5 diagonals. These diagonals intersect at F, G, H,

I, and J as shown. Using these 10 points, three sets of isosceles triangles (5 triangles in each set) can be formed so that:

(a) The five triangles in each set have the same shape and size and they do not have any overlap.

(b) Triangles from different set have the same shape but may or may not have the same size.

(c) The five triangles from one of these 3 sets can be rearranged to form a 5-pointed star.

Find: (1) Vertex angle in an isosceles triangle is the non-equal angle. From these three sets, list the three triangles that have A as the vertex angle.

(2) Draw the 5-pointed star using five triangles from (c).

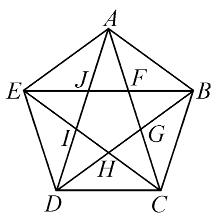

Fig. 1

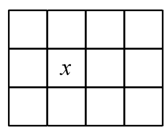

Fig. 2

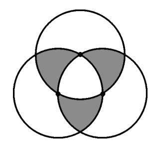
Fig. 3

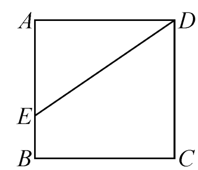

Fig. 4

7. As in the Fig. 2, there are $3 \times 4 = 12$ small squares and one of the squares has a letter of x in it. Using the squares in this figure, how many rectangles (not counting squares) can be formed that include the letter x?

8. The Fig. 3 shows three circles all with radius 10. If each circle goes through the centers of the other two circles, find the area of the shaded area. (Express your answer in terms of π.)

9. Among the proper factors of 324×325^2, how many are perfect squares?

10. Given a square $ABCD$ as in the Fig. 4. Draw a straight line from D to AB intersecting at E as in the figure. If the ratio of the area of $\triangle ADE$ to the area of quadrilateral $EDCB$ is $5:9$, find $\frac{BE}{EA}$ and express your answer in simplest fraction form.

11. Suppose the 6 digit number $\overline{18abc2}$ can be divided evenly by both 17 and 31. Find the largest possible value for $a+b+c$.

12. Suppose there is a pile of Black and white Go pieces. If 15 White pieces are taken away, the number of Black pieces to the number of White pieces is $2:1$. If another 45 Black pieces are further taken away, then the resulting ratio of remaining Black pieces to remaining White pieces is $1:5$. How many Black and White pieces were there in the beginning?

13. One travel agency has chartered a certain number of buses to accommodate the entire group of travelers. Suppose the maximum number of passengers each bus can hold is 32 and this travel agency wants each bus to hold exactly the same number of passengers. If the agency places 22 passengers in each bus, then there is one traveler got left out on the last bus. If the agency charters one less bus, then each bus would hold exactly the same number of travelers with no one left out after redistribution. How many buses did the agency originally charter and how many travelers are there?

14. A school has three classes and each class has the same number of students. The number of boys in the first class is the same as the number of girls in the second class, and the number of boys in the third class equals to $\frac{2}{5}$ of the total number of boys in the school. Among all the students in the school, what proportion of them are girls? (Express the answer in simplest proper fraction form.)

15. Suppose a, b, and c are natural numbers representing three sides of a triangle and two of these numbers are different prime numbers. If each of these three numbers satisfies conditions
$a+b+c=99$, $a+c=2b$, and $a<c$, how many such triangles are there?

16. Given there are n consecutive natural numbers starting with 1. Remove the 4 largest numbers in this group of n numbers. If the average value of the remaining numbers is 49, find n.

17. If the sum of the areas of two squares equals to the area of a third square, then we call these three squares forming an "elegant" set. Consider there are 20 squares each with different edge lengths equal to natural numbers from 1 to 20. How many "elegant" sets are there in these 20 squares?

18. There are 5 circular pieces each has a number from 1 to 5 placed as in the Fig. 5 in clockwise order. These pieces are moved according to the following instruction:
(1) Each piece must be moved in the clockwise direction.
(2) Move ① first and then move ②, ③, ④, ⑤.
Then repeat the process by moving ①, ②, ….
Move ① pass next one piece.
Move ② pass next two pieces.
Move ③ pass next three pieces.
Move ④ pass next four pieces.
Move ⑤ pass next five pieces.
And then repeat the process by moving ① pass next one piece and so on.
What is the order of these five pieces after 2012 moves? (Write the order with ① first.)

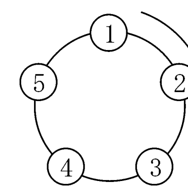

Fig. 5

19. The Fig. 6 shows a wooden cube that is consisted of $5\times5\times5$ small 1 cm^3 cubes. The shaded area on top, front, and side represent small cubes that are taken out all the way through to the opposite side of the wooden cube. Find the volume of the remaining part of this cube.

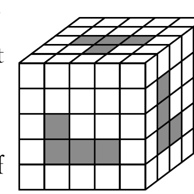
Fig. 6

20. Suppose n is a natural number and $S(n)$ represents the sum of the digits of n. If $n+S(n)=2013$, find n.

Team Round Answers

1. 2.
2. 2.
3. 110000_3, 324_{10}.
4. 18108.
5. 2.625.
6. (1) $\triangle AEI$, $\triangle AJF$, $\triangle AGB$.
(2) Fig. 1.

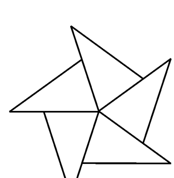
Fig. 1

7. 17.
8. 50π.
9. 36.
10. $\frac{2}{5}$.
11. 14.
12. 50 Black pieces and 40 White pieces.
13. 24 buses and 529 travelers.
14. $\frac{4}{9}$.
15. 3.
16. 101.
17. 6.
18. ①⑤④③②.
19. 79.
20. 1992, 2010.

Relay Round · Problems

First Round

1A. Suppose a box contains some red and white balls. The proportion of number of red to white balls is 12 : 35. If we take out $\frac{1}{3}$ of the red balls and $\frac{4}{7}$ of the white balls in the box, find the proportion of number of remaining red balls to remaining white balls. (Express the answer in simplest proper fraction form.)

1B. Let $\frac{n}{m}=$ TNYWR (The Number You Will Receive).

A rectangular piece of paper of size $n \times m$ can be rolled into various shapes of cylinders with no bases. What is the largest volume among these cylinders? (Express Answer in terms of π.)

Second Round

2A. Consider two sets of numbers: Set $A=\{1,3,5,7,\cdots,97,99\}$ and Set $B=\{2,4,6,8,\cdots,98,100\}$. If a number is randomly taken out of set A and another number from set B and calculate their sum, how many different sums are possible?

2B. Let $T=$ TNYWR (The Number You Will Receive).

Suppose an exam has 3 problems and the percentages of doing the first, second, and third problem correctly are $T\%$, 80%, 75%, respectively. If a student must have all these 3 problems correct to get an "A", what is the minimum percentage of students who get "A" in this exam?

Third Round

3A. If the sum of two 2-digit numbers $\overline{ab}$ and $\overline{ac}$ and a 1-digit number d is 30, and $d>b>c>a$, find $\overline{ab}$.

3B. Let $T=$ TNYWR (The Number You Will Receive).

Let a be the average of all the prime numbers that are larger than 10 and smaller than 20. Find the last digit of the product of $(a-T)$ by itself 2012 times.

Relay Round Answers

First Round

1A. $\frac{8}{15}$.

1B. $\frac{450}{\pi}$.

Second Round

2A. 99.

2B. 54%.

Third Round

3A. 13.

3B. 6.

Individual Round · Problems

First Round

1. Suppose the three squares in the Fig. 1 are same size with edge length of a. As in the figures, the first square has one shaded incircle, the second square has 9 equal size shaded incircles, and the third square has 16 equal size shaded incircles. Find the sum of all areas in those three squares that are not shaded.

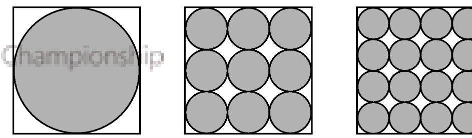

Fig. 1

2. Suppose $a = 17 + 20 + 23 + \cdots + 170$,

$b = 23 + 28 + 33 + \cdots + 218$,

$c = 1^2 + 2^2 + 3^2 + \cdots + 26^2$.

Rank the letters a, b, and c from the smallest to largest.

(**Note**: The formula $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ can be used to find c.)

3. Let M be a set that consists of the first 2012 positive integers and S is a subset of M. At least how many elements S must have to guarantee there exists one pair of numbers in S having their difference that is a multiple of 5?

4. If 2 red light bulbs and 4 yellow light bulbs are stringed together vertically to make a light signals of 6 light bulbs (different combination makes different signal), how many ways can they be stacked to form different light signals?

Second Round

5. If your watch shows it is 9 : 20 in the morning now, what time (hours and minutes) would it be after 17999998 minutes?

6. Consider three workers A, B, and C all making the same product. It is known that it would take A 4 hours and B 5 hours to make the same number of items. Also, it would take B 4 hours and C 3 hours to make the same number of items. Find how long it takes A to produce the same number items as what C produces in 15 hours?

7. If the sum of the first n natural numbers starting with 1 is a 3-digit number with identical digits, find n.

8. Find the smallest natural number a so that the remainder is 8 when the number of $43a$ is divided by 9.

Third Round

9. What is the maximum number of prime numbers that can be formed by using the first 9 natural numbers (from 1 to 9) as digit(s)? (Each number can be used only once.)

10. If the sum of 11 consecutive natural numbers is 1155, find the largest number among this group.

11. Suppose there are six goods with weights 7.5 kilograms, 5 kilograms, 4.5 kilograms, 4 kilograms, 4 kilograms, and 2.5 kilograms. They are to be placed in three boxes in a way that makes the heaviest box as light as possible. This way, what is the weight in the heaviest box?

12. Given that a mouse can run 5 steps in the time which a cat can run 3 steps but the distance for a cat's 4 steps is the same as a mouse's 7 steps. Now suppose the mouse has a 3 meters head start of the cat. What is the distance the cat must run before it can catch up to the mouse?

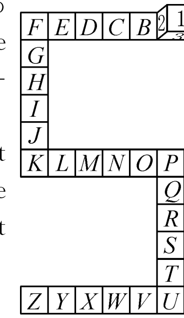

Fig. 2

Fourth Round

13. As in the Fig. 2, 26 English alphabets are written, in order, inside the 26 squares that arranged in the S shape. Consider a wooden cube with 6 faces each is written a number from 1 to 6 where 1 and 6 are on the opposite sides, 2 and 5 are opposite, and 3 and 4 are opposite. In the beginning, place the wooden cube on letter A with the number 1 on top and the number 2 facing letter B as shown in the figure. Now roll the cube along the S shape figure following the letter in order. What is the number on top when the cube is on the letter Z?

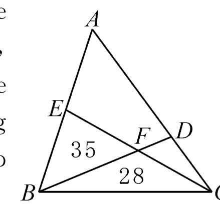

Fig. 3

14. As in the Fig. 3, point E bisects AB, point D is on AC such that $AD = 2DC$, and F is the intersection of CE and BD. If $S_{\triangle BEF} = 35$, and $S_{\triangle BFC} = 28$, find $S_{\triangle DCF}$.

FIfth Round

15. How many 3-digit odd numbers $\overline{abc}$, such that $a < b \leqslant c$ and $a + b + c = 21$?

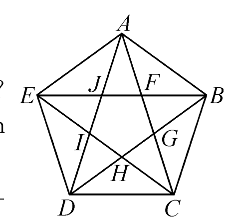

Fig. 4

16. The Fig. 4 is a regular pentagon with vertices A, B, C, D, and E. Each of its interior angles is 108°. Connect the vertices and form 5 diagonals. These diagonals intersect at F, G, H, I, and J as shown. Isosceles triangles can be formed by using A as a vertex and two points from the other 9 points. How many such isosceles triangles can be formed?

Individual Round Answers

First Round

1. $3a^2 - \frac{3\pi a^2}{4}$.
2. $b < a < c$.
3. 6.
4. 15.

Second Round

5. 9:18 in the morning.
6. 16.
7. 36.
8. 5.

Third Round

9. 6.
10. 110.
11. 10.
12. 63.

Fourth Round

13. 4.
14. $\frac{28}{3}$, 14, or no solution.

Fifth Round

15. 3.
16. 24.