2012 **World Mathematics Team Championship**
Intermediate Level
Team Round · Problems

World Mathematics Team Championship
www.wmtc.global

1. Suppose a, b, and c are non-zero natural numbers and the estimated value for $\frac{a}{4}+\frac{b}{8}+\frac{c}{16}$ is 5.3. Find its exact value in decimal.

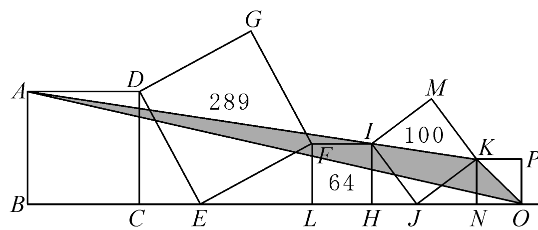

Fig. 1

2. Suppose we have 5 squares each has one of its sides or at least one of its vertices on a straight line as shown in Fig. 1 and two neighboring squares share one common vertex. If the middle three squares have areas of 289, 64, and 100, respectively, find the area of $\triangle AOK$.

3. As in Fig. 2, mini-robots M and N start walking at the same time with constant speed clockwise from a 6 meters by 6 meters square $ABCD$'s two vertices A and B, respectively. Suppose the speed of M is $\sqrt{2}$ times the speed of N and suppose M catches up to N for the second time after 20 minutes. How many meters N travels in one minute?

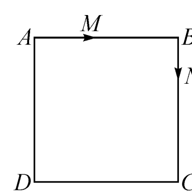

Fig. 2

4. Suppose a shop was selling some calculators at an original price which is a whole number less than \$200. Later, these calculators are on sales at 60% of the original price. If the shop has revenue of \$30,498 from selling a total of 200 of these calculators, how many were sold before the sales?

5. Let $\triangle ABC$ be an isosceles right triangle with C as its right angle and the length of its equal sides is 12 cm. Let CD be its height where D is on AB. Let E be a point on CD with $ED=6$ cm and let P be a point inside $\triangle ABC$ such that $\angle DPE$ is an acute angle. Find the area of the region in cm^2 where all possible P are located. (Express your answer in a whole number that is closest to the precise answer.)

6. Find the largest possible natural number n so that $\frac{n}{513-2n}$ is the square of an integer.

7. Among three people A, B, and C, A tells the truth 2 out of 3 times, B tells the truth 6 out of 7 times, and C tells the truth 4 times out of 5 times. If there is a statement in which both A and B say it is true statement but C says it is false, then what is the probability this is a true statement? (Express your answer in simplest fraction.)

8. Given

$$x+y+z=3,$$
$$x^2+y^2+z^2=7,$$
$$x^3+y^3+z^3=12.$$

Find xyz.

9. Solve the equation $8(2x^2+6x+6)(3y^2-3y+2)=15$ for (x, y).

10. Given that the edge lengths of $\triangle ABC$ are three distinct integers and its perimeter is 21. Among all such triangles, find the edge lengths of the triangle with the largest angle.

(Reference Formula: Law of Cosines $\cos C=\dfrac{a^2+b^2-c^2}{2ab}$, where a, b, and c are the edge lengths of $\triangle ABC$ and angle C is opposite to side c. Note that smaller number $\cos C$ means larger angle C.)

11. Solve the equation $2x+xy+\dfrac{x}{y}=243$ for all possible pairs of positive integers x and y.

12. Suppose positive integers x, y, and z satisfy system of equations $\begin{cases}xy+z=13\\ x+yz=23\end{cases}$. Find the smallest value for $x+y+z$.

13. Suppose we have one large 50 liter container that is fully filled with alcohol and two equal sized smaller empty containers. Now, we totally filled one of the smaller containers with liquid (all alcohol) from the large container and then re-fill the large container with pure water. We, again, totally filled the other smaller empty container with liquid from the large container. Refill the large container with pure water again. If the content of this large container now consists of 50% alcohol and 50% water, what is the volume of each of these two small containers?

14. As in Fig. 3, the segment DE divided the square $ABCD$ into a triangle $\triangle ADE$ and a trapezoid $EDCB$. If $S_{\triangle ADE}:S_{EDCB}=5:19$ ($S_{\triangle ADE}=$ area of $\triangle ADE$), find the proportion between the perimeter of $\triangle ADE$ and the perimeter of quadrilateral $EDCB$. (Express the proportion in simplest fraction form.)

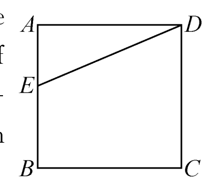

Fig. 3

15. Let Rt$\triangle ABC$ be a right triangle with $AB=BC$, and let P be a point inside the triangle. If $PA=5$, $PB=4$, and $PC=1$, find the area of $\triangle ABC$.

16. Suppose a rectangle's edge lengths are integers and its perimeter m is a multiple of 3. If $90\leqslant m\leqslant 100$, among rectangles that satisfy these conditions, find the area of the largest rectangle.

17. Among all the triangles that have integer edge lengths and a perimeter of 24, what are the edge lengths of the one with the largest area?

(**Note**: If the lengths of a triangle's three edges are a, b, and c, let $a+b+c=2l$, then the area of this triangle satisfies $S^2=l(l-a)(l-b)(l-c)$).

18. If the 4 largest prime numbers are taken out of the first n natural numbers (from 1 to n), then the average for the remaining natural numbers is $24\dfrac{3}{46}$. Find all possible n.

19. As in Fig. 4, a rectangular box $ABCD$-$EFGH$ is hanging up in the air with $AE=6$, $AB=7$, and $AD=8$. Let point M be the center of rectangle $ABFE$ and point N be the midpoint of the edge FG. Suppose a piece of candy is placed on M and on N and an ant is located at point D. What is the distance of the shortest path for the ant crawling along the surface of the box to get a candy?

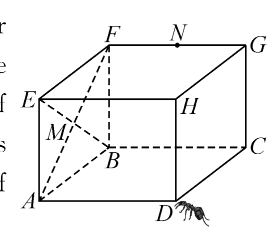

Fig. 4

20. Suppose the product of the digits of a 3-digit number plus 10 times the sum of the digits equals to the original 3-digit number. Find all possible 3-digit numbers that satisfy this property.

World Mathematics Team Championship
www.wmtc.global

Team Round Answers

1. 5.25 or 5.3125.
2. 153.
3. $\frac{3}{2}(\sqrt{2}+1)$.
4. 91.
5. 44.
6. 256.
7. $\frac{3}{4}$.
8. -2.
9. $\left(-\frac{3}{2},\frac{1}{2}\right)$.
10. 5, 6, 10.
11. $\begin{cases}x=24\\y=8\end{cases}$, or $\begin{cases}x=54\\y=2\end{cases}$.
12. 12.
13. $50-25\sqrt{2}$ liters.
14. $\frac{15}{22}$.
15. $\frac{17}{2}$.
16. 576.
17. 8, 8, 8.
18. 50.
19. $\sqrt{133.25}$.
20. 119, 166, 195, 379, 498, and 999.

World Mathematics Team Championship
www.wmtc.global
World Mathematics Team Championship
www.wmtc.global
World Mathematics Team Championship
www.wmtc.global
World Mathematics Team Championship
www.wmtc.global

Relay Round · Problems

First Round

1A. How many integers from 0 to 10 that will make the expression n^3+n^2+7 a prime number?

1B. Let T = TNYWR (The Number You Will Receive).

Suppose the perimeter of rectangle is $24+2T$ and suppose the shorter pair of its edges has a length of $T+3$. Let P be a point inside $ABCD$. What is the probability that $AP>T+3$? (Use $\pi=3$ and round your answer to nearest tenth.)

Second Round

1A. As in Fig. 2, any 3 squares configuration arranged as in Fig. 1 satisfies $A=B+C$. Find x.

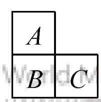

Fig. 1

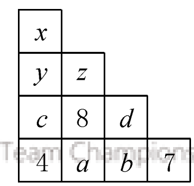

Fig. 2

2B. Let T = TNYWR (The Number You Will Receive).

Let T be the perimeter of an isosceles triangle $\triangle ABC$ with $AB=BC$ and $AC=5$. If D is a point on AB such that $AC=DC$, find AD.

Third Round

3A. Suppose there are three kinds of salt solutions.

Solution A: 35 liters with 8% concentration.

Solution B: 25 liters with 3% concentration.

Solution C: 30 liters with 11% concentration.

To mix these solutions into 50 liters of new solution with 7% concentration, what is the maximum amount of Solution C can be used?

3B. Let T = TNYWR (The Numbers You Will Receive).

Suppose the edge length of the square in Fig. 3 is m and its area is T. Find the area of the shaded region. (Express your answer in simplest fraction.)

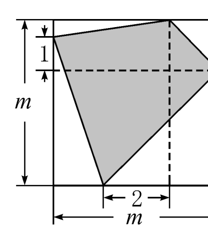

Fig. 3

Relay Round Answers

World Mathematics Team Championship
www.wmtc.global

First Round

1A. 4.

1B. 0.4.

Second Round

2A. 35.

2B. $\frac{5}{3}$.

Third Round

3A. 25.

3B. $\frac{27}{2}$.

Individual Round · Problems

World Mathematics Team Championship
www.wmtc.global

First Round

1. The irrational number $\sqrt{2+\sqrt{3+\sqrt{37}}}$ lies between which two consecutive integers?

2. If $\triangle ABC$ has edge lengths $2\sqrt{5}$, $3\sqrt{2}$, and $\sqrt{26}$, find the area of this triangle.

3. How many natural numbers n that can make $F(n)=\frac{1}{n}+\frac{2}{n}+\cdots+\frac{15}{n}$ an integer.

4. What is the remainder when 221^{2012} is divided by 9.

Second Round

5. A certain number of students decide to share their total travel cost equally. If each student pays \$28, then the total payment is \$18 short. If each pays \$30, then the total payment is \$4 too much. What is the total travel cost?

6. Let a, b, and h represent the length of an isosceles triangle's equal side, base, and height to one of the equal sides, respectively. Suppose a, b, and h are all natural numbers and a is 3 larger than b. And triangle's perimeter and area have the same numerical value. Find h.

7. Given a 45 meters round log with circumference of 3 meters. While this log rolls down 200 meters from a hill, a squirrel runs from one end of the log to the other end staying on the log the whole time. How long of a distance did this squirrel cover?

8. As in Fig. 1, AB and CD are perpendicular chords of circle O that are 3 and 2 units from the center of the circle, respectively. These two intersecting chords divided the circle into four parts with area S_1, S_2, S_3, and S_4 as shown. Find $(S_1+S_3)-(S_2+S_4)$.

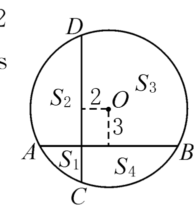

Fig. 1

Third Round

9. Let $ABCD$ be a square of unit length. Use A as vertex to make an equilateral triangle $\triangle AMN$, with M on BC and N on CD. Find the length of BM.

10. Suppose the hypotenuse of a right triangle has a length of $|x-3|$ and one of the other two sides has a length of $|4-3x|$. When the length of the third side is at a maximum, what is the perimeter of this triangle?

11. Suppose a is a non-zero simple irreducible fraction. If $a+\frac{15}{4a}$ is a positive integer, how many possible values can a be taken?

12. Given a sequence of numbers $7, 8, 11, \cdots$. Let a_n be the n^{th} number in this sequence. If

$$a_n+a_{n+1}+a_{n+2}+a_{n+3}=31,$$

find the sum of the first 2013 numbers of this sequence.

Fourth Round

13. Given an equilateral triangle $\triangle ABC$ with $AC = 20$ meters. Two micro-robots m and n start from vertices A and B, with the speed of 2 meters per minute and 3 meters per minute respectively, moving toward vertex C along the sides of $\triangle ABC$ and each arrives at points M (on AC) and N (on BC) after t minutes. If $MN = BN$ at that time, find the integer portion of time t.

(**Note**: Law of Cosines $\cos C = \frac{a^2+b^2-c^2}{2ab}$ where a, b, and c are the lengths of $\triangle ABC$ and angle C is opposite to side c. Larger angle C implies smaller $\cos C$.)

14. Given a square $ABCD$ of length 5. As shown in Fig. 2, circle M has a radius of 2 and tangents at DC and CB. Circle N tangents externally to circle M at point P and to DA and AB. If EF is these two circles' internal common tangent line through P intersecting at E and F on DA and AB, respectively, find the length of EF.

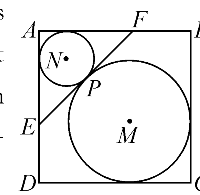

Fig. 2

Fifth Round

15. Suppose the shortest side of an obtuse triangle has a length of 10 and the other two sides have lengths of $2a+3$ and $3a+2$. If $a>0$, find the range of values for a.

16. If real numbers x, y, and z satisfy $x+\frac{1}{y}=\frac{3}{2}$, $y+\frac{1}{z}=\frac{7}{3}$, $z+\frac{1}{x}=4$, find the value for xyz.

Individual Round Answers

First Round

1. 2 and 3.
2. 9.
3. 16.
4. 7.

Second Round

5. \$ 326.
6. Such triangle doesn't exit.
7. 205.
8. 24.

Third Round

9. $2-\sqrt{3}$.
10. $\frac{5}{2}+\frac{5}{4}\sqrt{2}$.
11. 4.
12. 15600.

Fourth Round

13. 3.
14. $6\sqrt{2}-4$.

Fifth Round

15. $\sqrt{21}<a<11$.
16. 6 or $\frac{1}{6}$.

World Mathematics Team Championship
www.wmtc.global