2011 **World Mathematics Team Championship**
Junior Level
Team Round • Problems

1. Given four numbers with an average of 12. Suppose this average is 9 greater than the smallest of these four numbers, 5 less than the second largest number, and its sum with the second smallest number is same as the largest number. Write out these four numbers in ascending order.

2. The *Hope Cup Competition* started in 1990. This year is 2011. If we write $\frac{2011}{1990}$ as a continued fraction

$$a+\cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{d+\cfrac{1}{e}}}}.$$

(a) What is the largest natural number among the integers a, b, c, d, and e?

(b) What is the sum of a, b, c, d, and e.

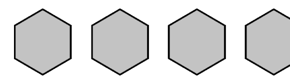


Fig. 1

3. Place four regular hexagons with edge 1 on a desk without overlapping as in the Fig. 1. Suppose they are allowed to touch each other matching completely side by side and form different figures. If L represents the perimeter of the forming figure, find all possible L.

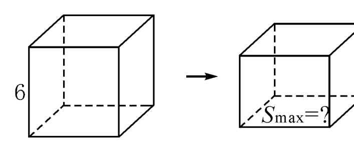


Fig. 2

4. As in the Fig. 2, given an open top cubic tin box (with only 5 sides) of dimensions 6cm × 6cm × 6cm. Suppose we want to change it into an open top box with natural number edges and with only $\frac{5}{6}$ of the original volume.

(a) Among all these possible boxes, find the largest base area in cm^2?

(b) When the base area is the largest, and the least material are used, find the sum of length width and height in cm.

5. As in the Fig. 3, a large rectangle is partitioned into 9 smaller rectangles. The numbers inside the small rectangle represent their areas. The numbers outside the small rectangles represent the lengths of the edges. Find the area of the original large rectangle.

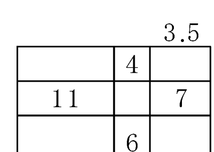


Fig. 3

6. As in the Fig. 4, $ABCDEFGH$ is a regular Octagon. If the area of isosceles trapezoid $ABCD$ is 12, find the area of this octagon.

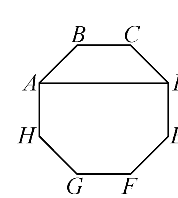


Fig. 4

7. Use a piece of paper that is 0.02cm thick, 87.92m long, and 0.12m wide to wrap tightly around a 0.12m long cylindrical tube of diameter 10cm in multiple layers. This forms a paper tube wrapping around the original cylinder. As in the Fig. 5, what is the thickness of the resulting paper tube in cm? (Use 3.14 for π)

8. A properly working clock has its hour hand and minute hand forming a 30° at 11 o'clock in the morning. When is the next time when this clock's hour and minute hands again form that same angle?

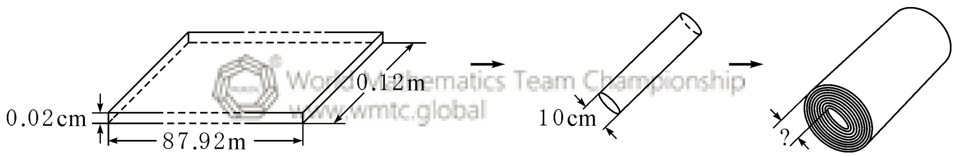


Fig. 5

9. What is the smallest prime number p so that $29p + 3$ is equal to the product of two consecutive non-zero natural numbers?

10. Draw two identical triangles totally inside a piece of rectangular paper so that no vertex or edge of either triangle would touch the edge of the paper. These two triangles would divide the paper into n regions.
What are all the possible value for n?

11. Let a, b, c, d be non-zero natural numbers that are not more than 4. Two of them are identical, and $(a+b)(b+c)(c+d)(d+a)=900$. Find $a+b+c+d$.

12. Suppose the sum of the digits of a 4-digit number is 4 and this number is still a 4-digit number if we reverse the order of its digits. Among all such possible 4-digit numbers, find the one that is closest to the perfect square of some 2-digit number.

13. A piece of 13×13 square paper is being cut up into many smaller pieces. If these pieces are putting back together to form 3 squares of different sizes and each edge is integer number. Find the sum of these 3 squares' perimeters.

14. As in the Fig. 6, $ABCD$ is a rectangle, $AB=6$, $AD=10$. If point E is on the diagonal AC, $AE=2EC$, and $S_{\triangle AEF}=\frac{1}{7}S_{ABCD}$, find $AF : FD$.

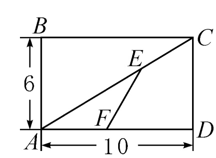


Fig. 6

15. How many pairs of natural numbers x and y satisfying $3x+5y=121$?

16. A company sold 30% of a certain merchandise from its inventory for \$150 each, and each lost 5%. Now, a customer paid x dollars each buying up the remaining inventory. This transaction results the company making a total of 5% profit on the whole inventory. Find the ratio of x to the cost of each item.

17. Let n be any non-zero natural number and m is the product of $n-1$, n, $n+1$, and $n \times n+1$.
(a) Is m a multiple of 6?
(b) Is m a multiple of 5?

18. What is the remainder when $8^{2011} - 3^{2011} - 6^{2011}$ is divided by 10?

19. A square field $ABCD$ has 32 holes as in the Fig. 7 (9 holes on each side). A worker carries 32 flags and placing them into these holes clockwise. He starts with Hole A and skips 5 holes before he places another flag in the next hole. After he has been around the square field a number of times, he discovers that there are still n holes without a flag. Find n.

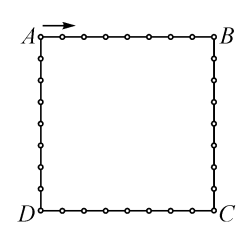


Fig. 7

20. Let N be the largest number of regions that can be formed by drawing 2011 straight lines on a plane. Find the sum of all digits of N.

Team Round Answers

1. 3, 8, 17, 20.

2. (a)94; (b)104.

3. 24, 22, 20, 18, 16, 14.

4. (a)180cm^2; (b)28cm.

5. 77.

6. 48.

7. 4.

8. 11 o'clock 54 $\frac{6}{11}$ Minutes.

9. 3.

10. 2, 3, 4, 5, 6, 7, 8.

11. 11.

12. 1021.

13. 76.

14. 3 : 4.

15. 8.

16. $\frac{153}{140}$.

17. (a) Yes; (b) Yes.

18. 9.

19. 16.

20. 20.

Relay Round · Problems

First Round

1A. The edge lengths of a rectangle are prime numbers and its perimeter is 60. Find the largest area of such rectangle.

1B. Let T = TNYWR (The Number You Will Receive).
As in the Fig. 1, D is on the line segment CG and the sum of the areas of square $ABCD$ and square $CEFG$ is T. If the sides of these two squares differ by 1, find the area of quadrilateral $AEFG$.

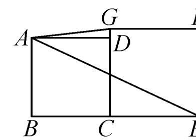


Fig. 1

Second Round

2A. Given that a, b, c, d are natural numbers and a is equal to the area of a square P. If $a < b + c$, $b < 2c$, $c < 3d$, $d < 500$, then which one of the four numbers 268, 272, 276, 280 is the perimeter of P?

2B. Let T = TNYWR (The Number You Will Receive).
If the sum of the squares of three consecutive natural numbers is T-23, then what is the sum of these three consecutive natural numbers?

Third Round

3A. A team of 5000 some (more than 5000 but less than 6000) athletes is gathered in a football field. These athletes can either be grouped into several arrays of 12×12 or several arrays of 18×18. How many athletes are there?

3B. Let T = TNYWR (The Number You Will Receive).
Suppose T' is the number derived by reversing the digits of T and $T + T' + 1$ is the square of a certain natural number. If this natural number is the sum of 5 consecutive natural numbers, find these 5 consecutive natural numbers.

Relay Round Answers

1A. 221.
2A. 268.
3A. 5184.
1B. 121.
2B. 27.
3B. 18, 19, 20, 21, and 22.

Individual Round • Problems

First Round

1. In a certain year, there are exactly 4 Wednesdays and 4 Sundays in January. Which day of the week is February 14^{th} in that year?

2. Both aeroplane A and aeroplane B can fly for 6000 kilometers when they have a full tank of fuel. If both of the aeroplanes depart from Airport C with a full tank and aeroplane A can refuel aeroplane B from its own tank at some point in the air. In order to make sure both aeroplane A and B can return to Airport C before their fuel run out, what is the longest distance aeroplane B can reach (in kilometers)?

3. Consider the line segment AB as in the Fig. 1.

   If $CD=\frac{2}{5}AD$, $DB=\frac{2}{5}AB-1$, and $AB=12$, find CB.

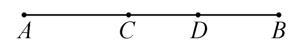


Fig. 1

4. If the digits in a 3-digit number $\overline{abc}$ satisfy $c(a+c)=40$ and $a(a+b)=36$, find all possible $\overline{abc}$.

Second Round

5. From 130 to 1300, inclusive, there are n natural numbers that are multiples of 17 or 71. Find n.

6. A company had x college-graduate-employees, which is 45% of the total employees. Now, another 120 college-graduate-employees are hired, making the percentage of college-graduate-employees reaches 75%. How many employees the company had before this latest hiring?

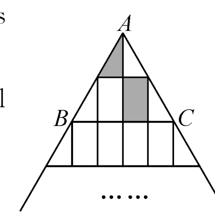
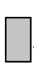


Fig. 2

7. As in the Fig. 2, $\triangle ABC$ is composed with identical right triangles  and rectangles . From top down, its first layer is composed with 2 right triangles. Its 2^{nd} layer is composed with 2 right triangles and 2 rectangles, and so on. Each right triangle has an area of 1 and each rectangle has an area of 2. Which layer has an area of 8042?

8. As in the Fig. 3, suppose the base AB of a parallelogram $ABCD$ is 8cm long and its height is 4cm. If the area of triangle BEF is 6cm^2 larger than the area of CDF, find the length of BE in cm.

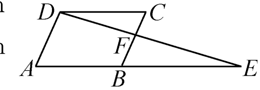


Fig. 3

Third Round

9. Find the number of factors of the natural number 371250 that are not larger than 11. Also, how many of these factors are prime numbers?

10. If a is the largest prime number less than 1000, then what is the smallest non-zero natural number b so that $a+b$ is also a prime number?

11. Place 2011 of the number 2011 side by side and form a (4×2011)-digit number. Find the remainder when this number is divided by 8.

12. As in the Fig. 4, $ABCD$ is a square with edge as $2a$. Let $\odot O$ be the largest circle inside square $ABCD$ and let $EFGH$ be the largest square inside $\odot O$. If S_1, S_2, and S_3 are used to represent the area of $\triangle EOF$, the area of the arch-shaped region EmF, and the area of curved region EBF, respectively. Find $S_1 : S_2 : S_3$ (Express the answer in terms of π).

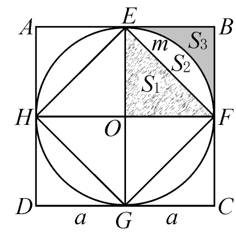


Fig. 4

Fourth Round

13. *Steven* stands on a bridge AB. His distance from A is $\frac{7}{16}$ of the distance AB. A train is coming toward A at a speed of 80 km/h. *Steven* has two choices. He can either run toward A and he will meet the train at A, or he can run toward B at the same speed then the train will catch up him at B. Find his speed in km per hour.

14. In a 12-hour clock, what time between 1 : 00 am and 2 : 00 am when the hour hand and the minute hand form a straight line?

Fifth Round

15. As in the Fig. 5, $\triangle A_1B_1C_1$, $\triangle A_2B_2C_2$ and $\triangle A_3B_3C_3$ are equilateral triangles. Let D_1 trisects A_1C_1 and bisects A_2B_2. And let D_2 trisects A_2C_2 and bisects A_3B_3. Also, the bases of all three triangles are all on a straight line. If the area of $\triangle A_1B_1C_1$ is 54cm^2, then find the area of the figure that is formed by the zigzag line segment $B_1A_1D_1A_2D_2A_3C_3B_1$.

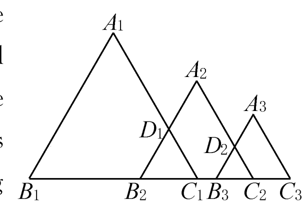


Fig. 5

16. A webcam, a DVD drive, and a hard disk drive together cost \$100. Suppose one DVD drive costs more than two webcams, two hard disk drives cost more than seven DVD drives, and eight webcams cost more than one hard disk drive. Assuming the cost of each of these items is a whole number of dollars, how much does each item cost?

Individual Round Answers

First Round

1. Saturday.

2. 4000 kilometers.

3. 7.08.

4. 604 or 395.

Second Round

5. 85.

6. 100.

7. 2011.

8. 11mm.

Third Round

9. 8, 4.

10. 12.

11. 3.

12. $2 : (\pi - 2) : (4 - \pi)$.

Fourth Round

13. 10km/h.

14. $1 : 38\frac{2}{11}$ am. and $1 : 05\frac{5}{11}$ am.

Fifth Round

15. 80cm^2.

16. Webcam \$9.00, DVD Drive \$20.00, Hard Disk Drive \$71.00.