2010 **World Mathematics Team Championship Intermediate Level**

Team Round • Problems

1. $\sqrt{2011\sqrt{2010\sqrt{2009\times 2007+1}+1}+1}=$ ______.

2. If a and b are rational numbers and $x=\dfrac{\sqrt{5}+1}{2}$ is a solution for the equation $x^3-ax-b=0$, then $a=$ ______ and $b=$ ______.

3. Given that real numbers a, b and c satisfy $a-b=\dfrac{7}{4}ab$ and $b-c=\dfrac{7}{4}bc$. When a takes on values of, in order, 1, $\dfrac{2009}{2010}$, $\dfrac{2008}{2010}$, $\cdots$, $\dfrac{3}{2010}$, $\dfrac{2}{2010}$, $\dfrac{1}{2010}$, 0, $-\dfrac{1}{2010}$, $-\dfrac{2}{2010}$, $-\dfrac{3}{2010}$, $\cdots$, $-\dfrac{2008}{2010}$, $-\dfrac{2009}{2010}$, -1, b and c would take on a total of ______ negative numbers as values.

4. Given that x, y, z are integers. If $x>y>z$ and $2^x+2^y+2^z=4.625$, then $xyz=$ ______.

5. As shown in Fig. 1, $\odot O$ and the hypotenuse of $\text{Rt}\triangle AOB$ intersect at C and D. If C and D trisect AB and the radius of $\odot O$ is 5, then $AB=$ ______.

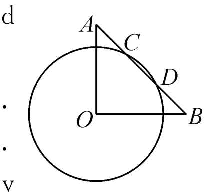

Fig. 1

6. A store is purchasing a product to sell. This product costs the store \$10 each. If the store sells this product for \$15 each, then it can sell 120 of this product. In general, if the sale price goes up by $a\%$, then the sales will also go down by $a\%$. To get the largest profit, the store should set the sale price at \$ ______ each.

7. Suppose that the proportion of copper to iron is $1:2$ in alloy A of 2 kg, and $3:2$ in alloy B of 7 kg. If we are going to melt some alloy A and alloy B together and form a new alloy so that this new alloy has a proportion of copper to iron as $6:5$, then the maximal amount of this new alloy we can form is ______ kg.

8. Given a and b are 1-digit numbers, and $\overline{45ab}$ is a 4-digit number. To make the absolute difference between the two numbers $27(a+b)^2$ and $\overline{45ab}$ as small as possible, use $a=$ ______ and $b=$ ______.

9. Suppose $f(x)$ is a polynomial of x. If $f(x)$ has a remainder of 3 when it is divided by $2(x-1)$ and $2f(x)$ has a remainder of -4 when it is divided by $3(x+2)$, then $3f(x)$ has a remainder of ______ when it is divided by $4(x^2+x-2)$.

10. As it is shown in Fig. 2, point P is a point inside the regular hexagon $ABCDEF$. Use this point to divide this hexagon into 6 triangles $\triangle PAB$, $\triangle PBC$, $\triangle PCD$, $\triangle PDE$, $\triangle PEF$, $\triangle PFA$ with six areas S_1, S_2, S_3, S_4, S_5, S_6, respectively. If $S_1-S_2+S_3=1$, then $S_3+S_6=$ ______.

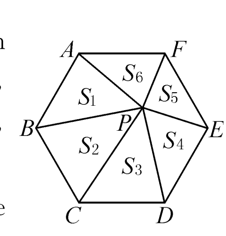

Fig. 2

11. Let P be the product of 3 consecutive odd positive integers. Then the largest integer that can divide into all such P is ______.

12. Suppose a, b, c are the lengths of a triangle's three sides. Also, a is an integer and it is also the largest of the three lengths. If a, b, c satisfy the set of equations

$$a^2 + 2b - 10c + 10 = 0 \quad ①$$

$$a + 2b - 5c + 7 = 0 \quad ②$$

then $a =$ ______, $b =$ ______, $c =$ ______.

13. As it is shown in Fig. 3, if E and F are points on the square $ABCD$'s sides BC and CD, respectively, and $\angle EAF = 45°$, then the smallest possible value for $\frac{EF}{AB}$ is ______.

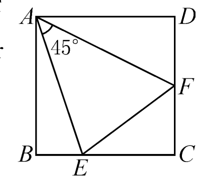

Fig. 3

14. Suppose the straight line $y = (1-k)x + k\ (k < 1)$ intersects the hyperbola $y = \frac{6}{x}$ at $A(x_1, y_1)$ in the 1st Quadrant and at $B(x_2, y_2)$ in the 3rd Quadrant. If we draw perpendicular lines from A and B to the x-axis and they intersect at points M and N, respectively, then the area of the quadrilateral $AMBN$ has the smallest value of ______ when $k =$ ______.

15. Consider the 5×5 square grid in Fig. 4. ______ squares can be formed by using some of these 36 grid points as vertices.

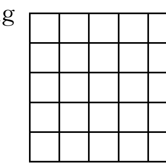

Fig. 4

16. The roots of the set of equations

$$\begin{cases} x_1 + x_2 = x_2 + x_3 = \cdots = x_{2010} + x_{2011} = 1 \\ x_1 + x_2 + x_3 + \cdots + x_{2010} + x_{2011} = 2011 \end{cases},$$

are ______.

17. If $a + b + c = 0$, then $\left(\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c}\right)\left(\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b}\right) =$ ______.

18. As in Fig. 5, isosceles right triangle ABC has its hypotenuse AC on the straight line MN. Its sides BA and BC both have length of 3. If we rotate this triangle, without sliding, to the right along the line for one revolution until AC is on MN again, then point A travelles a distance of ______ during this rotation.

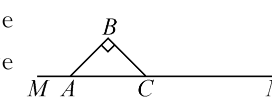

Fig. 5

19. Consider the circle $\odot O$ in Fig. 6. The diameter of $\odot O$ is $AB = 20$. Point P is outside $\odot O$ and PC and PB are tangents to $\odot O$ at C and B, respectively. Chords $CD \perp AB$ at E. PA intersects CD at M. If $\frac{AE}{EB} = \frac{1}{4}$, then the area of $\triangle PCM$ is ______.

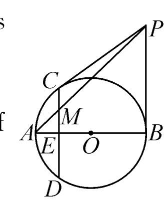

Fig. 6

20. Suppose there are 6 cities A, B, C, D, E, F and 6 islands a, b, c, d, e, f with ferries connecting them. Each city must have at least one ferry connection to one island. If the total number of connections from each city A, B, C, D, E is 5, 4, 3, 2, 2, respectively,
and if the total number of connections from each island a, b, c, d, e is 4, 3, 2, 1, 1, respectively, then the total number of connection lines to cities from island f is ______.

Team Round Answers

1. 2010.
2. 2; 1.
3. 1722.
4. 6.
5. $3\sqrt{10}$.
6. 20.
7. 8.8.
8. 6; 7.
9. $5x+4$.
10. 2.
11. 3.
12. 4; 2; 3.
13. $2\sqrt{2}-2$.
14. 0; 12.
15. 105.
16. $x_1=x_3=\cdots=x_{2009}=x_{2011}=1006$; $x_2=x_4=\cdots=x_{2008}=x_{2010}=-1005$.
17. 9.
18. $\frac{3(2+3\sqrt{2})}{4}\pi$.
19. 32.
20. 6.

Relay Round · Problems

First Round

1A. It took 24 hours for a piece of wooden log floating downstream on a river from Location A to Location B. A boat has a speed of 18 km/hour when the water speed is 0. This boat can go from A to B in a time that is $\frac{3}{5}$ of the time it took the boat to go from B to A. Then the distance between Location A and Location B is ________ km.

1B. Let $A =$ the answer passed from your teammate. Rearrange the digits from the number $2A$ which is a 3-digit number and form new numbers. If S is the sum of numbers from all the possible rearrangements and we also know that $S = \left[\frac{n(n+1)}{2}\right]^2 - 3(a+b+c)$ where a, b, c are the digits of $2A$ and n is a natural number, then $n =$ ________.

Second Round

2A. If $x < 0$, $y < 0$ and $x - 6y = -\sqrt{xy}$, then $\frac{x}{y} =$ ________.

2B. Let $b =$ the answer passed from your teammate. Suppose $x^2 - y^2 + ax + by + 1$ can be factored into linear factors of x and y. Then $a =$ ________.

Third Round

3A. If three line segments of lengths $\sqrt{n}, 1 + \sqrt{n+2}$ and $\sqrt{n} + \sqrt{n(n+2)}$ are used to form a triangle, then the possible natural numbers $n =$ ________.

3B. Let $m =$ the largest number of the answer passed from your teammate. Find the three interior angles of the triangle that has $\sqrt{m}, 1 + \sqrt{m+1}$, and $\sqrt{m} + \sqrt{m(m+1)}$ as the lengths of its three sides.

Relay Round Answers

First Round

1A. 108.

2B. 9.

Second Round

2A. 9.

2B. $\pm\sqrt{85}$.

Third Round

3A. 1, 2.

3B. $15°, 30°, 135°$.

Individual Round · Problems

First Round

1. The sum of all the coefficients of the expanded expression $(xy+2x^2y^2+3x^3y^3+4x^4y^4+5x^5y^5)^2$ is ________.

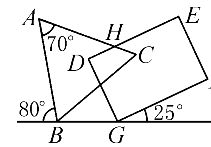

Fig. 1

2. There are ________ pairs of integers (m, n) that satisfy $m^2-n^2=625$.
3. As in Fig. 1, $AB=AC$ and $DEFG$ is a rectangle. Then $\angle DHC=$ ______.
4. As in Fig. 2, an equiangular polygon with an even number of sides. Let A_1A_2 be the first side, A_2A_3 be the second side, and so on. If the fifth side is parallel to the first side, then the degree of one of its interior angles is ________.

Second Round

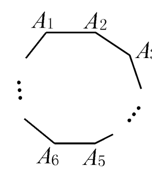

Fig. 2

5. There are ________ possible acute triangles with perimeter not more than 2010 and the lengths of its sides are consecutive natural numbers.
6. If $f(x)=|x^2-9\cdot 10^{499}x-10^{999}|$, then $f(11)$ has ________ different factors among the numbers 1,2,3,4,5,6,8,9,10,11,12.
7. Given points $A(1,1)$, $B(-1,2)$ and point C is on the straight line $y=-x$, then the shortest perimeter for triangle ABC is ________.
8. Given a and b are 1-digit numbers, and $\overline{58ab}$ is a 4-digit number. To make the absolute difference between $27a^b$ and $\overline{58ab}$ as small as possible, $a=$ ________, $b=$ ________.

Third Round

9. Given $\sqrt{x}=\dfrac{1-a}{2}$ (a is a constant). Simplify $\sqrt{x+a}-\sqrt{x-a+2}=$ ________.
10. The whole number 12345…9899100 has a remainder of ________ when it is divided by 9.
11. For $n=1,2,3\cdots$, how many of the following choices represent a number that cannot be divided by 6 evenly?

    ① n^3-n. ② $8n^3-2n$. ③ $2n^3+3n^2+n$. ④ n^5-n.

12. As shown in Fig. 3, E is on the diagonal BD of a 1×1 square $ABCD$ and $BE=BC$. P is a point on CE and $PQ\perp BC$ at ponit Q, $PR\perp BE$ at point R. Then $PQ+PR$ has a total length of ________.

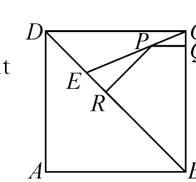

Fig. 3

Fourth Round

13. Suppose each of $x_1, x_2, \cdots, x_n$ can take on one of three numbers $-1, 2$ and -3. If $x_1 + x_2 + \cdots + x_n = 3$ and $x_1^2 + x_2^2 + \cdots + x_n^2 = 15$, then $x_1^5 + x_2^5 + \cdots + x_n^5 =$ ________.

14. A point in an x-y coordinate system is called a lattice point with integer coordinates. If the vertices of a convex n-sided polygon are all lattice points and its interior or sides do not contain any other lattice point, then $n =$ ________.

Fifth Round

15. Insert numbers $1, 2, 3, \cdots, 9$ into the 9 circles of Fig. 4, respectively, so that the sum of the numbers in the three circles in each side of triangle ABC and triangle DEF are all equal to 18. There are a total of ________ ways to fill those 9 numbers.

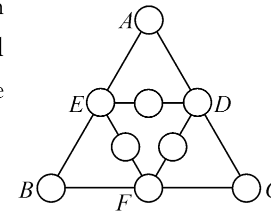

Fig. 4

16. As in Fig. 5, the area of triangle ABC is 1. D and E are trisection points for side BC. F and G are trisection points for side AC. AE and BF intersect at H. Then the area of the quadrilateral $CEHF$ is ________.

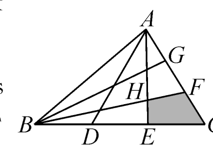

Fig. 5

Individual Round Answers

First Round

1. 225.

2. 10.

3. $145°$.

4. $135°$.

Second Round

5. 666.

6. 2.

7. $3+\sqrt{5}$.

8. 6; 3.

Third Round

9. $-2, a-1$.

10. 1.

11. 0.

12. $\frac{\sqrt{2}}{2}$.

Fourth Round

13. 93.

14. 3, 4.

Fifth Round

15. 6.

16. $\frac{1}{6}$.